

**The Performance of User Verification Using Two  
Fingerprint Based On Error Rate**

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## **Abstrak**

Teknologi biometrik, utamanya cap jari, telah mula mendapat perhatian pengguna dalam memastikan keselamatan sesuatu tempat atau harta benda. Dalam implementasi sistem pengesahan individu menggunakan satu cap jari pada masa sekarang, sistem telah berhadapan dengan pelbagai masalah seperti cap jari kotor dan pengguna tidak meletakkan cap jari dengan betul. Apabila masalah yang berlaku hanya menjejaskan sebahagian kecil permukaan cap jari, ianya akan dapat diselesaikan pada fasa pemprosesan imej dan pengecaman corak. Apabila keadaan sebaliknya berlaku, maka pendekatan yang lain terpaksa digunakan. Oleh itu, pendekatan menggunakan dua cap jari pada proses pengesahan telah digunakan bagi mengatasi masalah ini. Dengan tidak hanya merujuk kepada penggunaan ibu jari yang selalu digunakan, potensi pada jari yang lain telah cuba dikenal pasti bagi mendapatkan dua jari terbaik, yang telah digunakan dalam proses ini. Beberapa kaedah mudah dalam pengkelasan keputusan daripada kedua-dua cap jari ini telah dicuba, dan kaedah yang terbaik telah digunakan untuk melihat tahap prestasi sistem berbanding dengan sistem yang berasaskan satu cap jari. Dalam kajian ini, kadar jumlah ralat telah digunakan sebagai penentu tahap prestasi sistem. Walaupun tidak dapat menyelesaikan masalah yang dihadapi sepenuhnya, jumlah kadar ralat bagi sistem pengesahan ini telah dapat dikurangkan menggunakan pendekatan yang telah dicadangkan.

## **Abstract**

Biometric technology, especially fingerprint, attract users around the world to use it to secure their places or properties. On the current implementation of fingerprint based person verification, the system face several problems such as noisy finger and fingerprint misplacement by the user. When the problem only effected a small part of fingerprint, it is solved or minimized on the image processing and pattern recognition phase. But, when the problem involves a larger part of fingerprint, another approach needs to be used. To solve or minimize this kind of problem, the approach uses two fingerprints on the verification process have been experimented on this research. By not only referring to the thumbs which are usually used, the potential of the other fingerprint have been studied to find the two best fingerprints as used in this process. Using several simple methods to classify the decisions from both fingerprints; the best classifier have been used to study the performance level of the system compared to the current system used in single fingerprint. In this study, total error rate has been used as an indicator to the performance level of the system. Although the problems have not been totally solved, but the total error rates for fingerprint verification system has been minimized by using this approach.

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## **List of abbreviations**

3D - Three dimension

AIDC – Automated Identification and Data Capture Center

ATM – Auto Teller Machine

BLS – Bureau of Labor Statistics

BWG – Biometric Working Group

CCD - Charge-Coupled Device

DCOM – Distributed Component Object Model

DLL – Dynamic Link Library

DNA - Deoxyribonucleic acid

FAR – False Acceptance Rate

FRR – False Rejection Rate

GMPC - Government Multi-Purpose Card

IBG – International Biometric Group

ID – Identity

ISR - Intelligent System Report

SDK – Software Development Kit

TER – Total Error Rate

UID – User Identity

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# **Chapter 1**

## **Introduction to the research**

### **1.1 Introduction**

There are many applications or processes used to prove the identity of a person. It covers non-critical e-mail application such as web-based email to the most critical access control such as in the defense agency. All need the best method to prove the user identity. Most of them still use simple identification or verification modules such as login and password.

In the context of system security, verification is referring to the process of comparing identity of a user against the single stored sample of the identity (IPC, 1999). It has played a major role in our life everyday. The system that a person has to interact with need to know who the person is before it can give access or right to the person. It also needs to decide whether the person should get the right to get to the system or not. Besides, it also needs to know whether the person that gets to the system is really the person that he claims to be. In addition, security of the important information on an organization also depends on the reliability of the verification system that controls the access of people or staff in the organization, building or computers. Thus, the verification process has been an important part of many applications to prove the identity of the user.

The contents of  
the thesis is for  
internal user  
only

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